Open dynamical systems with a moving hole

Beibei Sun
Joint work with Prof. Derong Kong and Zhiqiang Wang

Chongqing University

Numeration and Substitution 2025, September 9, 2025

Introduction

Given a finite set of $d \times d$ real matrices $\mathscr{M} = \{M_1, \ldots, M_r\}$, Rota and Strang introduced the concept of *joint spectral radius*

$$\hat{\rho}(\mathscr{M}) := \lim_{n\to\infty} \sup_{i_1\ldots i_n \in \{1,2,\ldots,r\}^n} \|M_{i_1}\cdots M_{i_n}\|^{1/n},$$

which extends the notion of spectral radius of a single matrix. The set $\mathscr{M}$ is said to satisfy the *finiteness property* if there exists a finite block $i_1 \ldots i_n \in \{1,2,\ldots,r\}^n$ such that

$$\hat{\rho}(\mathscr{M}) = \rho\left(M_{i_1}\cdots M_{i_n}\right)^{1/n},$$

where $\rho(M)$ denotes the spectral radius of a matrix M.

Introduction

- Let $b \geq 3$ and $m \geq 2$ be integers, and define $D_b := \{0, 1, \ldots, b-1\}$. For any $\omega \in D_b^m$, let $G_\omega = (V, E)$ be a directed graph , where $V = D_b^{m-1}$ and $(\mathbf{u}, \mathbf{v}) \in E$ for $\mathbf{u} = u_1 \ldots u_{m-1}, \mathbf{v} = v_1 \ldots v_{m-1} \in V$ if $u_2 \ldots u_{m-1} = v_1 \ldots v_{m-2}$ and $u_1 \ldots u_{m-1} v_{m-1} \neq \omega$.
- Let A_ω be the corresponding adjacency matrix of G_ω, whose size is $b^{m-1} \times b^{m-1}$.

Result

Theorem 1

The set $\mathscr{A} = \left\{A_\omega : \omega \in D_b^m\right\}$ *has the finiteness property. Furthermore, for any periodic and progressively overlapping sequence* $\omega = (\omega^0\omega^1 \ldots \omega^{n-1})^\infty \in \left(D_b^m\right)^{\mathbb{N}}$ *we have*

$$\hat{\rho}(\mathscr{A}) = \rho(A_{\omega^0}A_{\omega^1} \cdots A_{\omega^{n-1}})^{1/n} = \lambda,$$

where $\lambda \in (b-1, b)$ *is a Pisot number satisfying the equation* $x^m - (b-1)(x^{m-1} + x^{m-2} + \cdots + x + 1) = 0$.

Thank you for attentions!